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Scanning-Phase Correction for an Offset-Feed Near-Field Gregorian Reflector

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A method is presented to correct the scanning-phase error of an offset-feed near-field Gregorian reflector antenna. A numerical search is performed to find the array scan angle for each element to produce a specified reflector scan angle. This results in a nonlinear array-aperture phase distribution that produces a linear phase front across the main reflector's aperture. For the geometrical parameters considered, the phase correction can be accomplished without any loss in main-beam power or scan range.

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I. Introduction

The greatest advantage of a phased-array antenna is its capability to scan a single beam or multiple beams over a broad angular region. The main disadvantage is that to produce a small beamwidth, a large aperture and thus a large number of array elements are required. The number of elements is determined by the spacing required to avoid grating lobes in the field of view. A method used to reduce the number of elements required is to magnify the size of the array optically until the beamwidth requirement is satisfied. This is done by using the array as the source feed to a reflector system [1-3]. Unfortunately, the maximum scan angle and the grating lobe separation of the combined system are both inversely proportional to the magnification. Therefore, there are practical limits to the amount of magnification allowed. As the array is scanned off the boresight direction, optical abberations occur that distort the magnified image of the array. The optical path length and the actual angle at which radiation from the image points leaves the system are different for each point. This leads to phase error, which reduces the peak gain of the antenna.

The scanning capability of an offset-feed near-field Gregorian reflector is limited primarily by phase errors across the image aperture. A linear phase across the array plane produces a nonlinear phase across the image plane, which increases with the array scan angle. Since the reflector is fed by a phased array, it is possible to compensate for optical abberations by adjusting the relative phase of the signal supplied to each array element [4-7].

The purpose of this study is to demonstrate a method of correcting the scanning-phase error of a near-field offset-feed Gregorian reflector system. This method views the reflector as a simple magnification system that images each array element from a point on the array plane to a point on the reflector aperture plane. Thus, the reflector aperture plane is the same as the image-

array plane. Requiring a linear phase across the image array implies that the angle of each ray leaving the image array aperture is the same. Thus, a ray leaving each antenna element in the array plane is traced through the reflector to the image aperture-plane to find the final exit angle. In a perfect optical system, that angle should be inversely proportional to the magnification of the system. If it is not, then the array-plane scan angle is adjusted until the exit angle matches the theoretical value. This produces a scan phase for each array element, together with a phase that matches each optical path length to correct the phase distortion in the image aperture-plane. Higher-order effects such as diffraction and mutual coupling are not considered for this analysis.

II. Principle of Reflection

The principle of reflection is somewhat misleading because it is based on human perception rather than on an understanding of the physics. It is "esthetic" to view a light ray as something that bounces from surface to surface. However, since light is part of the electromagnetic spectrum, it is well known that this does not adequately explain the phenomenon. An incident electric field (light ray) impinging on a surface induces volume and surface currents to flow in the object, which in turn produces a secondary source of radiation. This process takes place so fast that to the human eye, it appears that the light simply bounces from the surface.

Designs for reflecting antennas are derived from the optical properties of parabolic, hyperbolic, and elliptical reflecting surfaces. Using optical design methods is valid when the diameter of the reflector is many times that of the wavelength of the radiation. The total electric field vector $\bar{E}^T$, representing the interaction of a light ray with a conducting surface, can be thought of as the sum of an incident vector field $\bar{E}^i$ and a reflected vector field $\bar{E}^r$, such that

$$\bar{E}^T = \bar{E}^i + \bar{E}^r. \tag{1}$$

The incident and reflected electric fields can be decomposed into a tangential and normal component with respect to the reflecting surface as

$$\bar{E}^i = \bar{E}^i_n + \bar{E}^i_t \tag{2}$$

$$\bar{E}^r = \bar{E}^r_n + \bar{E}^r_t. \tag{3}$$

The boundary conditions on the total electric field at the surface of a perfect conductor are given by

$$\hat{n} \times \bar{E}^T = 0 \tag{4}$$

$$\hat{n} \cdot \bar{E}^T = \frac{\rho_s}{\epsilon} \tag{5}$$

where ρ_s is the surface charge on the conductor and ϵ is the permittivity of the medium. Applying eq. (4) to eq. (1) yields the relationship between the tangential incident and reflected fields as

$$\bar{E}_t^r = -\bar{E}_t^i. \tag{6}$$

This implies that the incident and reflected tangential fields have equal magnitudes but opposite signs at the reflector surface. The principle of conservation of energy may be applied to reason that the magnitude of the incident field and the magnitude of the reflected field must be equal at the conductor surface. This then implies that the normal components must also have equal magnitudes at the conductor surface. It is well-known that the surface charge on a perfectly conducting surface illuminated by an incident wave is nonzero, and therefore the normal components of the incident and reflected fields must have the same magnitude and direction at the conductor surface. The relationship between the incident and reflected electric fields may be expressed in vector form as

$$\hat{n} \times \bar{E}^r = -(\hat{n} \times \bar{E}^i) \tag{7}$$

$$\hat{n} \cdot \bar{E}^r = \hat{n} \cdot \bar{E}^i. \tag{8}$$

Taking the cross product of eq. (7) with $\hat{n}$ and using the vector identity

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B}) \tag{9}$$

yields the expression

$$\bar{E}^r = 2(\hat{n} \cdot \bar{E}^i)\hat{n} - \bar{E}^i \tag{10}$$

commonly known as Snell's law of reflection.

Let the propagation direction of the incident and reflected fields be denoted by the unit vectors $\hat{s}^i$ and $\hat{s}^r$, respectively. Snell's law implies that the angle between the incident wave and the surface normal is the same as the

angle between the reflected wave and the normal. This implies that

$$\hat{n} \times \hat{s}^r = \hat{n} \times \hat{s}^i \tag{11}$$

$$\hat{n} \cdot \hat{s}^r = -(\hat{n} \cdot \hat{s}^i) \tag{12}$$

which then leads to the relation

$$\hat{s}^r = \hat{s}^i - 2(\hat{n} \cdot \hat{s}^i)\hat{n}. \tag{13}$$

This equation can be put in the form

$$s_m^r = \sum_{n=1}^{3} T_{mn} s_n^i \tag{14}$$

where T_{mn} represents the reflection-matrix coefficient taken from the matrix equation

$$\begin{bmatrix} s_x^r \\ s_y^r \\ s_z^r \end{bmatrix} = \begin{bmatrix} 1-2n_x^2 & -2n_x n_y & -2n_x n_z \\ -2n_x n_y & 1-2n_y^2 & -2n_y n_z \\ -2n_x n_z & -2n_y n_z & 1-2n_z^2 \end{bmatrix} \begin{bmatrix} s_x^i \\ s_y^i \\ s_z^i \end{bmatrix} \tag{15}$$

Thus, the reflection properties of a perfectly conducting surface can be computed once the surface normals are found.

III. Offset-Feed Near-Field Gregorian Design

The general form of a classical Gregorian optical reflector system is shown in Fig. 1. Note that a spherical wave emanating from focus 1 of the ellipse reflects off the elliptical subreflector and is focused at focus 2. The spherical wave emanating from focus 2 then reflects off the parabolic surface of the main reflector, producing a plane wave. A typical antenna system would have a horn either receiving or radiating at focus 1. The subreflector is assumed to be in the far-zone field of the radiating source. Note that the subreflector blocks a portion of the reflector aperture; this blockage is inherent in this axially symmetric design and leads to a loss in gain and an increase in the sidelobe level.

If the spherical-wave point source is replaced by a two-dimensional array of point source radiators, then even though the subreflector is in the far zone of each radiating element, it is in the near zone of the array. The wave emanating from the array can be approximated by a plane wave. Given this assumption, the elliptical subreflector must be replaced by a parabolic subreflector so that the plane wave produced by the array can be converted into a spherical wave by the subreflector. If the array is laterally offset, then it is advantageous to use an offset geometry such as that shown in Fig. 2, to avoid any blockage. This offset configuration uses two confocal paraboloidal reflector sections. The array configuration chosen is shown in Fig. 3. The geometry is composed of a hexagonal lattice truncated to form an approximately circular aperture.

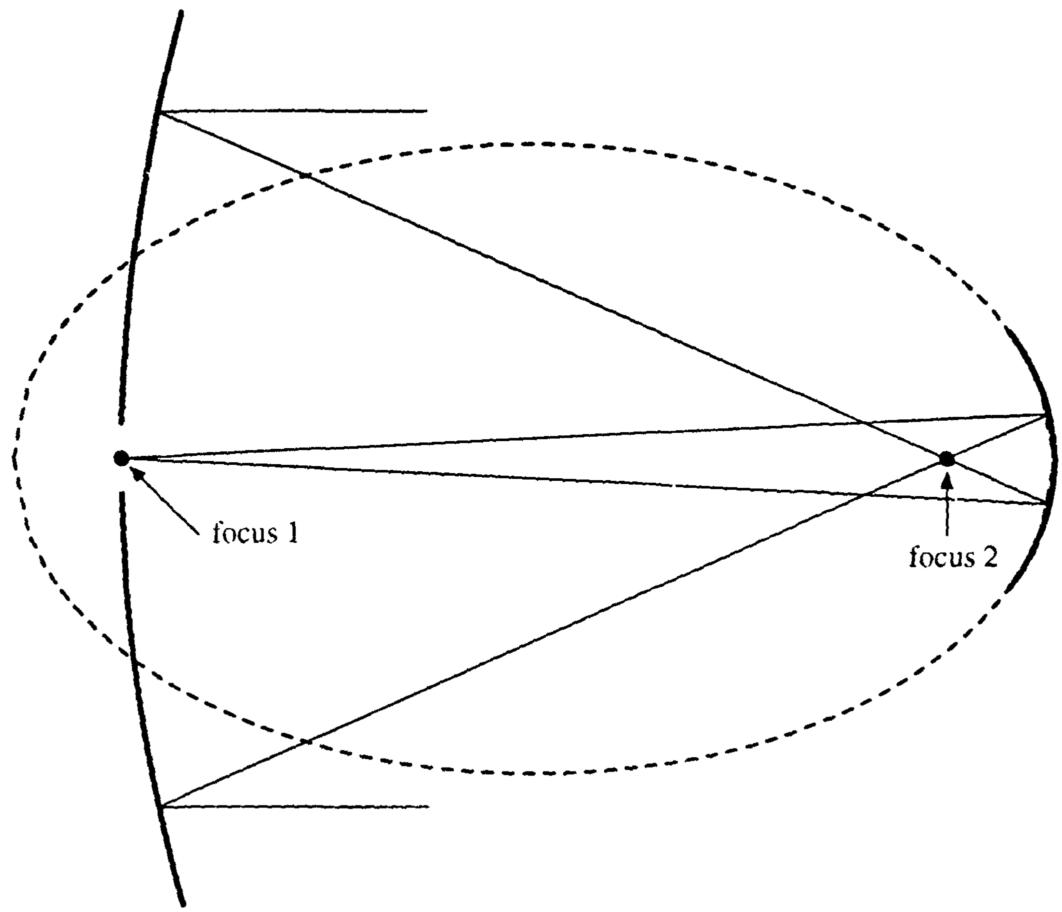

Fig. 1. Gregorian Dual Reflector Antenna

Assume the array is located in the plane (x, y, z_a). The equation of the subreflector is given by

$$z = z_s - \frac{1}{4F_s}(x^2 + y^2) \tag{16}$$

and the equation of the main reflector is

$$z = z_m + \frac{1}{4F_m}(x^2 + y^2). \tag{17}$$

Let $\bar{R}$ denote the position vector on the surface of either reflector given by

$$\bar{R} = x\hat{x} + y\hat{y} + z\hat{z}. \tag{18}$$

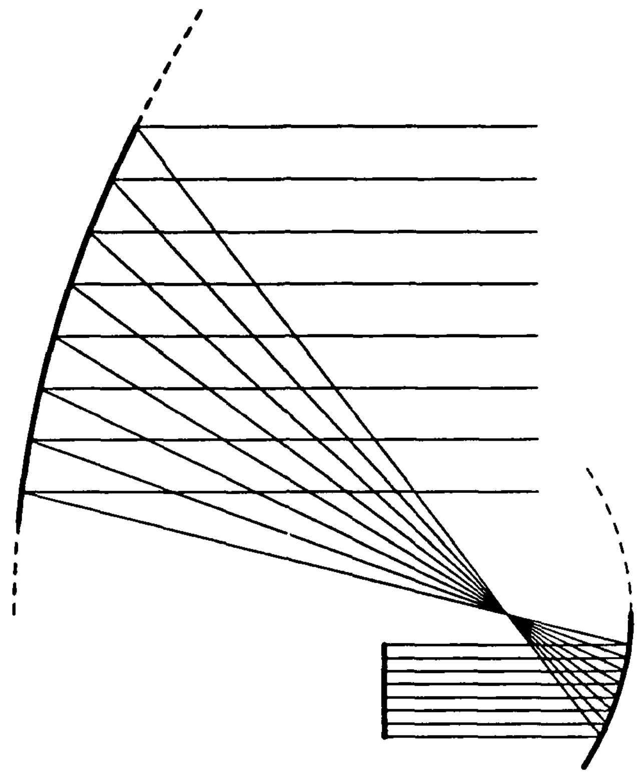

Fig. 2. Offset Gregorian Dual-Reflector Antenna

A unit vector that is normal to the surface may now be defined by the relation

$$\hat{n} = \pm \frac{\left(\frac{d\bar{R}}{dx} \times \frac{d\bar{R}}{dy}\right)}{\left|\frac{d\bar{R}}{dx} \times \frac{d\bar{R}}{dy}\right|} \tag{19}$$

which yields the subreflector unit normal $\hat{n}_s$, given by

$$\hat{n}_s = \frac{-x\hat{x} - y\hat{y} - 2F_s\hat{z}}{\sqrt{x^2 + y^2 + 4F_s^2}} \tag{20}$$

and the main reflector unit normal $\hat{n}_m$, given by

$$\hat{n}_m = \frac{-x\hat{x} - y\hat{y} + 2F_m\hat{z}}{\sqrt{x^2 + y^2 + 4F_m^2}}, \tag{21}$$

where the sign is chosen such that the angle between the normal and the reflected ray is acute.

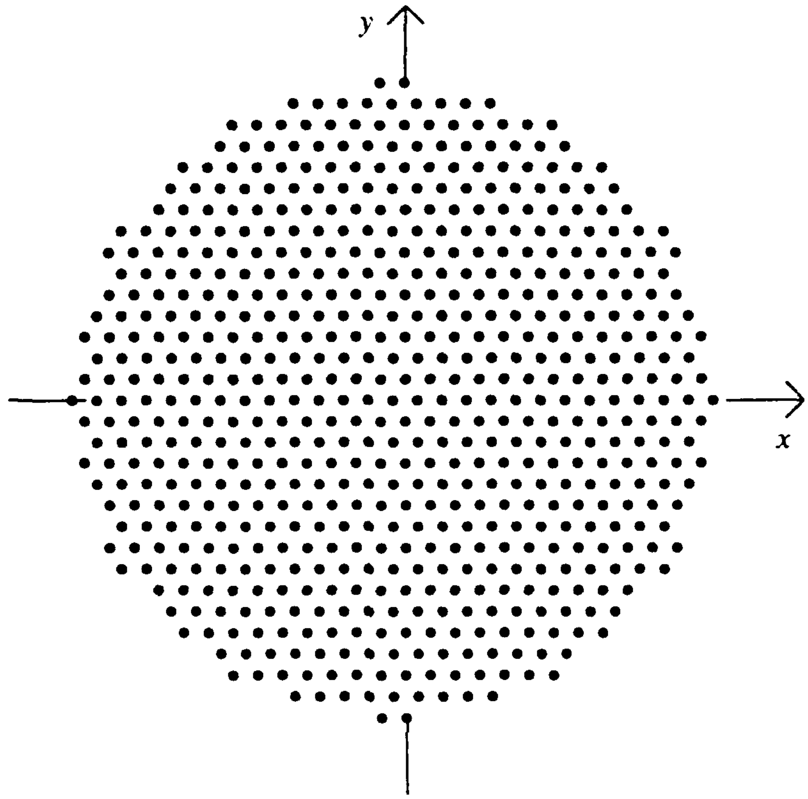

Fig. 3. Hexagonal Array with 613 Elements

Assume that the array is scanned only in the $x-z$ plane such that a scan angle α can be defined by the relation

$$\tan(\alpha) = \frac{x_1 - x_0}{z_1 - z_0} \tag{22}$$

where the angle is positive when measured counterclockwise from the positive z axis in the $x-z$ plane. A vector from the array plane to the subreflector surface can be defined as

$$\bar{R}_{01} = (x_1 - x_0)\hat{x} + (z_1 - z_0)\hat{z}. \tag{23}$$

Substituting eqs. (16) and (22) into eq. (23) yields

$$\bar{R}_{01} = \left[z_s - z_a - \frac{1}{4F_s}(x_1^2 + y_1^2)\right](\tan(\alpha)\hat{x} + \hat{z}). \tag{24}$$

Equating the x components of eq. (24) yields the quadratic equation in x_1,

$$A_1 x_1^2 + B_1 x_1 + C_1 = 0 \tag{25}$$

where

$$A_1 = \tan(\alpha) \tag{26}$$

$$B_1 = 4F_s \tag{27}$$

$$C_1 = -4x_0 F_s + (y_1^2 - 4(z_s - z_a)F_s)\tan(\alpha). \tag{28}$$

This equation will have only one feasible solution for x_1. The coordinates (x_1, y_1, z_1) of the ray intercept on the subreflector are then given by

$$x_1 = \begin{cases} x_0 & \alpha = 0 \\ \frac{-B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1} & \alpha \neq 0 \end{cases} \tag{29}$$

$$y_1 = y_0 \tag{30}$$

$$z_1 = z_s - \frac{1}{4F_s}(x_1^2 + y_1^2). \tag{31}$$

Each coordinate (x_1, y_1, z_1) can then be checked to make sure it intercepts the physical subreflector.

A general ray between the subreflector and the main reflector can be defined as

$$\bar{R}_{12} = (x_2 - x_1)\hat{x} + (y_2 - y_1)\hat{y} + (z_2 - z_1)\hat{z} \tag{33}$$

where (x_2, y_2, z_2) is the unknown point on the main reflector surface. An application of Snell's law on the surface of the subreflector shows that this ray must be parallel to the ray $\bar{S}$ defined by

$$\bar{S} = \bar{R}_{01} - 2(\hat{n}_s \cdot \bar{R}_{01})\hat{n}_s. \tag{34}$$

In order for eqs. (33) and (34) to be parallel, the slopes measured in two orthogonal planes must be equal. This leads to the two equations

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{S_y}{S_x} \tag{35}$$

$$\frac{z_2 - z_1}{x_2 - x_1} = \frac{S_z}{S_x} \tag{36}$$

where

$$S_x = T_{s11}R_{x1} + T_{s12}R_{y1} + T_{s13}R_{z1} \tag{37}$$

$$S_y = T_{s21}R_{x1} + T_{s22}R_{y1} + T_{s23}R_{z1} \tag{38}$$

$$S_z = T_{s31}R_{x1} + T_{s32}R_{y1} + T_{s33}R_{z1}. \tag{39}$$

Combining eqs. (35) and (36) yields a quadratic equation in x_2 given by

$$A_2x_2^2 + B_2x_2 + C_2 = 0 \tag{40}$$

where

$$A_2 = 1 + u_1^2 \tag{41}$$

$$B_2 = 2u_1(y_1 - u_1x_1) - 4u_2F_m \tag{42}$$

$$C_2 = (y_1 - u_1x_1)^2 + 4F_m(z_m - z_1 + u_2x_1) \tag{43}$$

$$u_1 = \frac{S_y}{S_x} \tag{44}$$

$$u_2 = \frac{S_z}{S_x}. \tag{45}$$

The coordinates (x_2, y_2, z_2) of the ray intercept on the main reflector are then given by

$$x_2 = \begin{cases} x_1 & \alpha = 0, x_0 = y_0 = 0 \\ \frac{-B_2 \pm \sqrt{B_2^2 - 4A_2C_2}}{2A_2} & \alpha \neq 0 \end{cases} \tag{46}$$

and

$$y_2 = y_1 + (x_2 - x_1)u_1 \tag{47}$$

$$z_2 = z_m + \frac{1}{4F_m}(x_2^2 + y_2^2) \tag{48}$$

The sign ambiguity in eq. (46) is due to the ambiguity in the ratio of the slopes. The correct sign is chosen by making sure that

$$\mathrm{sign}[x_2 - x_1] = \mathrm{sign}[S_x] \tag{49}$$

$$\mathrm{sign}[y_2 - y_1] = \mathrm{sign}[S_y]. \tag{50}$$

In a manner similar to that for the subreflector, each coordinate (x_2, y_2, z_2) can be checked to make sure it intercepts the physical surface of the main-reflector.

The general ray between the main reflector and the aperture plane is given by

$$\bar{R}_{23} = (x_3 - x_2)\hat{x} + (y_3 - y_2)\hat{y} + (z_3 - z_2)\hat{z} \tag{51}$$

Applying Snell's law on the surface of the main reflector yields

$$\frac{y_3 - y_2}{z_3 - z_2} = v_1 \tag{52}$$

$$\frac{x_3 - x_2}{y_3 - y_2} = v_2 \tag{53}$$

where

$$v_1 = \frac{T_{m21}R_{x1} + T_{m22}R_{y1} + T_{m23}R_{z1}}{T_{m11}R_{x1} + T_{m12}R_{y1} + T_{m13}R_{z1}} \tag{54}$$

$$v_2 = \frac{T_{m31}R_{x1} + T_{m32}R_{y1} + T_{m33}R_{z1}}{T_{m11}R_{x1} + T_{m12}R_{y1} + T_{m13}R_{z1}}. \tag{55}$$

Solving eqs. (52) and (53) yields the coordinate point (x_3, y_3, z_3) of the interception of the ray and the aperture plane of the main reflector; this point is given by

$$z_3 = z_p \tag{56}$$

$$y_3 = y_2 + (z_3 - z_2)v_1 \tag{57}$$

$$x_3 = x_2 + (y_3 - y_2)v_2 \tag{58}$$

where z_p is a specified aperture coordinate. The total path length of a ray P_l is the total length from the array plane to the aperture plane and is computed from the formula

$$P_l = |\bar{R}_{01}| + |\bar{R}_{12}| + |\bar{R}_{23}|. \tag{59}$$

IV. Small Scan-Angle Transfer Function

The reflector scan angle θ has a complicated nonlinear dependence on the array scan angle α. However, a simple linear transfer function can be derived for small α angles. This simple relationship will be used in the correction process to estimate the location of the main beam. The derivation is accomplished by taking a single axial ray, tracing it through to the aperture plane, and then making small angle approximations.

Reflection from the subreflector is illustrated in Fig. 4. The normal unit vector can be found by first noting that the family of curves of the subreflector can be written as

$$f(x,z) = z - \frac{x_1^2}{4F_s} = \text{positive constant}. \tag{60}$$

The inward normal vector is defined by

$$\hat{n}_s = -\frac{\nabla f}{|\nabla f|} \tag{61}$$

which yields

$$\hat{n}_s = \frac{\frac{x}{2F_s}\hat{x} - \hat{z}}{\sqrt{1 + \left(\frac{x}{2F_s}\right)^2}}. \tag{62}$$

It is useful to define an angle β between the unit vector $\hat{n}_s$ and the horizontal vector $-\hat{z}$ at the point of reflection (x_1, z_1), which may be represented as

$$\beta = \tan^{-1}\left(\frac{x}{2F_s}\right). \tag{63}$$

Now define ξ as the angle between the reflected ray and the unit vector $-\hat{z}$ such that $\xi = 2\beta + \alpha$.

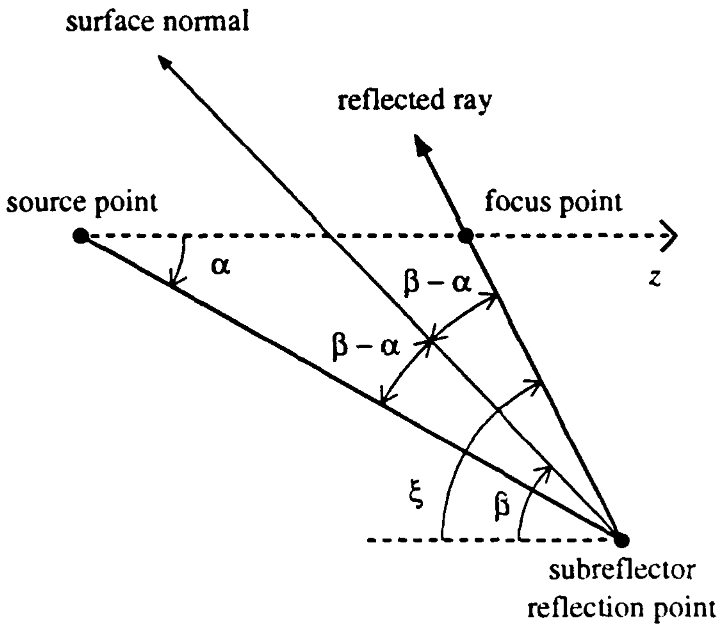

Fig. 4. Reflection from Subreflector

Figure 5 illustrates reflection from the main reflector surface. The coordinates of a ray interception on the main reflector surface are given by

$$\tan\xi = \frac{x_2 - x_1}{z_1 - z_2} \tag{64}$$

which leads to the quadratic equation

$$A_2 x_2^2 + B_2 x_2 + C_2 = 0 \tag{65}$$

where

$$A_2 = \frac{\tan(\xi)}{4F_m} \tag{66}$$

$$B_2 = 1 \tag{67}$$

$$C_2 = (F_s - F_m - \frac{x_1^2}{4F_s})\tan(\xi) - x_1. \tag{68}$$

The coordinates of the ray intercept on the main reflector are given by

$$x_2 = \begin{cases} x_1 & \xi = 0 \\ \frac{-B_2 - \sqrt{B_2^2 - 4A_2C_2}}{2A_2} & \xi \neq 0 \end{cases} \tag{69}$$

A unit vector normal to the main reflector pointing in the direction of the subreflector is given by

$$\hat{n}_m = \frac{\frac{x}{2F_m}\hat{x} + \hat{z}}{\sqrt{1 + \left(\frac{x}{2F_m}\right)^2}}. \tag{70}$$

Then defining an angle γ between the unit vector $\hat{z}$ and $\hat{n}_m$ at the reflection point (x_2, z_2) yields the relation

$$\gamma = \tan^{-1}\left(\frac{x}{2F_m}\right). \tag{71}$$

Defining an angle θ as the angle that the ray reflected off the main reflector makes with the unit vector $\hat{z}$, such that $\theta = \xi - 2\gamma$, it follows that

$$\tan(\theta) = \frac{x_3 - x_2}{z_3 - z_2} \tag{72}$$

which can be rewritten in terms of the aperture-plane x coordinate as

$$x_3 = x_2 + (z_3 - z_2)\tan(\theta). \tag{73}$$

The reflector scan angle θ and the array scan angle α are related as

$$\theta = \alpha + 2(\beta - \gamma). \tag{74}$$

For small scan angles this relationship may be approximated as

$$\theta \approx f(0) + f'(0)\alpha \tag{75}$$

where

$$f(\alpha) = \alpha + 2[\beta(\alpha) - \gamma(\alpha)] \tag{76}$$

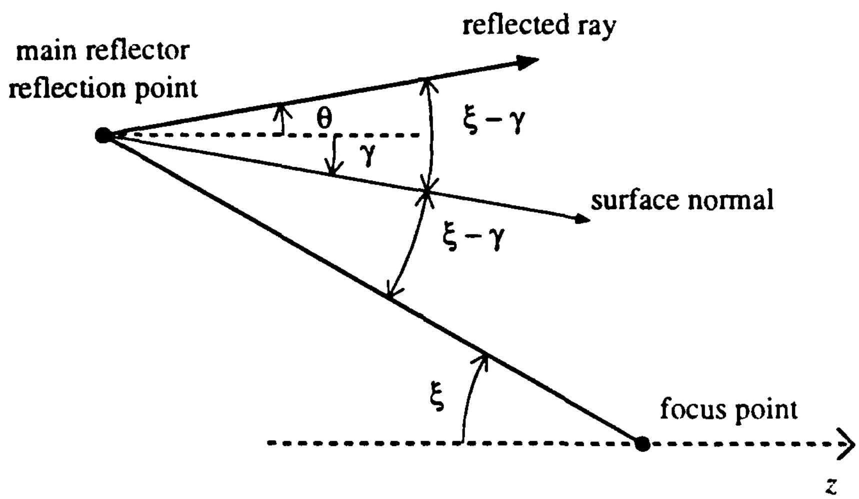

Fig. 5. Reflection from Main Reflector

By observation it can be seen that $f(0) = 0$ such that

$$\theta \approx f'(0)\alpha \tag{77}$$

where

$$f'(0) = 1 - 2\left(\left.\frac{d\gamma}{d\alpha}\right|_{\alpha=0} - \left.\frac{d\beta}{d\alpha}\right|_{\alpha=0} \right). \tag{78}$$

Using eq. (71), it can be shown that

$$\frac{d\gamma}{d\alpha} = \frac{2F_m}{4F_m^2 + x_2^2}\left(\frac{dx_2}{d\alpha}\right). \tag{79}$$

Using implicit differentiation on eq. (65) yields

$$\frac{dx_2}{d\alpha} = -\frac{\frac{dA_2}{d\alpha}x_2^2 + \frac{dC_2}{d\alpha}}{2A_2x_2 + B_2} \tag{80}$$

where

$$\frac{dA_2}{d\alpha} = \frac{1 + 2\frac{d\beta}{d\alpha}}{\cos^2(\alpha + 2\beta)} \tag{81}$$

$$\frac{dC_2}{d\alpha} = \left(F_s - F_m - \frac{x_1^2}{4F_s}\right)\frac{1 + 2\frac{d\beta}{d\alpha}}{\cos^2(2\beta + \alpha)} - \left(1 + \frac{x_1}{2F_s}\tan(2\beta + \alpha)\right)\frac{dx_1}{d\alpha} \tag{82}$$

Evaluated at $\alpha = 0$ yields

$$\left.\frac{d\gamma}{d\alpha}\right|_{\alpha=0} = \frac{1}{2} + \left.\frac{d\beta}{d\alpha}\right|_{\alpha=0} - \frac{1}{2}\frac{F_s}{F_m} \tag{83}$$

Substituting this into eq. (78) yields

$$f'(0) = \frac{F_s}{F_m}. \tag{84}$$

Therefore, for small array scan-angles, the array scan angle α and the reflector scan angle θ are related as

$$\theta = \frac{F_s}{F_m}\alpha. \tag{85}$$

This indicates that the maximum reflector scan angle is inversely proportional to the magnification of the system. This implies that the scanning capability of a reflector system is directly related to the magnification.

V. Small Scan-Angle Ray-Bundle Characteristics

The side view of the near-field Gregorian system with a boresight beam is shown in Fig. 6. The array has a diameter of $D_a = 18\lambda_0$. A hexagonal array of 613 elements fits within the diameter, with a center-to-center element spacing of $s = .6923\lambda_0$. The array is positioned in the $x-y$ plane at $z = 0$ with a horizontal offset position of $x_a = -9\lambda_0$. The subreflector and main reflector have focal lengths of $F_s = 24\lambda_0$ and $F_m = 96\lambda_0$, for a magnification of $M = 4$. The on-axis offsets are given by $z_s = 42\lambda_0$ and $z_m = -78\lambda_0$. The subreflector offset limits are given by $x_b = -40\lambda_0$ and $x_c = 6.2\lambda_0$. The main-reflector offset limits are given by $x_d = 15\lambda_0$ and $x_e = 110\lambda_0$. The rays are terminated at the reflector aperture plane located at $z_p = -46.49$. For this scan angle, all rays pass through the focus and form an exact point source. The front view of the ray bundle is shown in Fig. 7. Each array point of the source is uniformly magnified and reflected across the center of the array. There is no optical distortion.

The side view of the uncorrected ray bundle for $\alpha = -27°$ is shown in Fig. 8. The focus point moves below the axis and is blurred. Each ray comes out of the main-reflector aperture plane at a different angle but near the design angle of $\theta = 6.75°$. The front view of the ray bundle is shown in Fig. 9. Note that the equivalent area of the array is compressed in the vertical direction. This will increase the beamwidth slightly in the vertical plane. For the applications of interest in this study, the increased beamwidth is not considered critical.

The side view of the uncorrected ray bundle for $\alpha = 20°$ is shown in Fig. 10. The focus point moves above the axis and is blurred. Each ray comes out of the main reflector aperture plane at a different angle but near the design angle of $\theta = -5°$. The front view of the ray bundle is shown in Fig. 11. Note that the equivalent area of the array is expanded in the vertical direction. This will decrease the beamwidth slightly in the vertical plane.

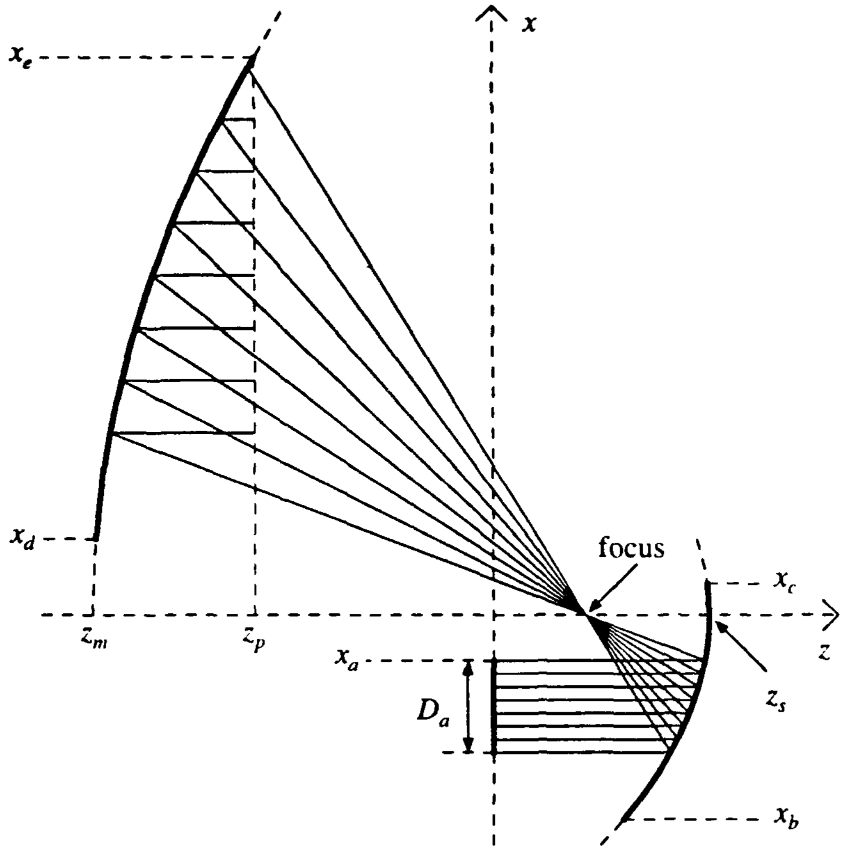

Fig. 6. Side View of Ray Bundle with $\alpha = 0°$

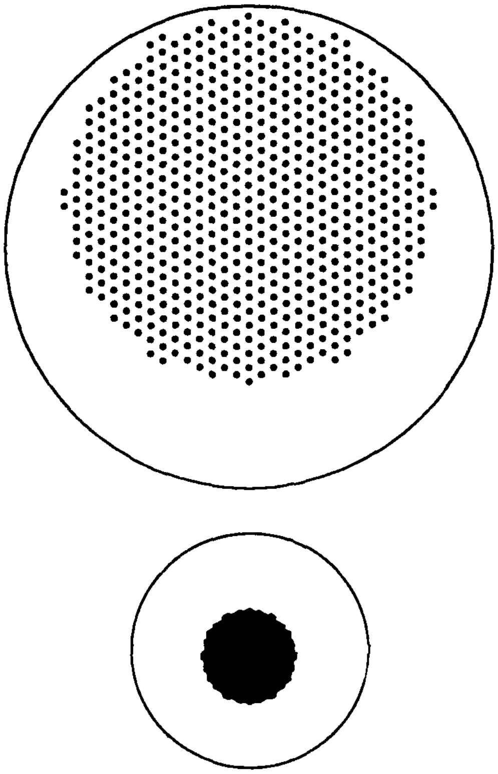

Fig. 7. Front View of Aperture Plane Ray Bundle with $\alpha = 0°$

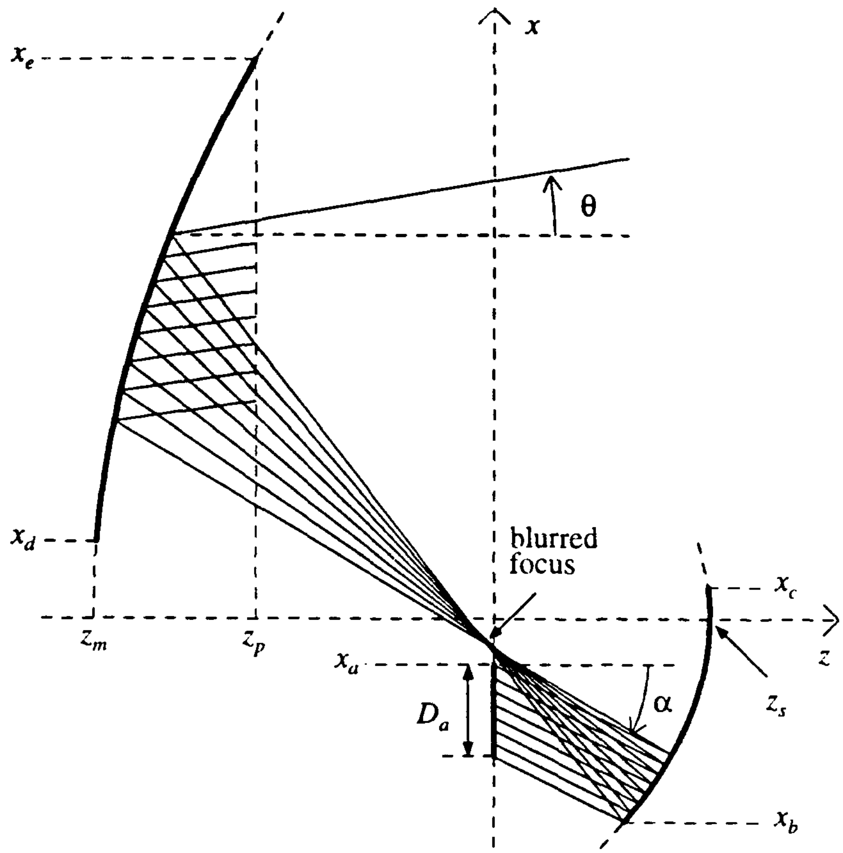

Fig. 8. Side View of Ray Bundle with $\alpha = -27°$

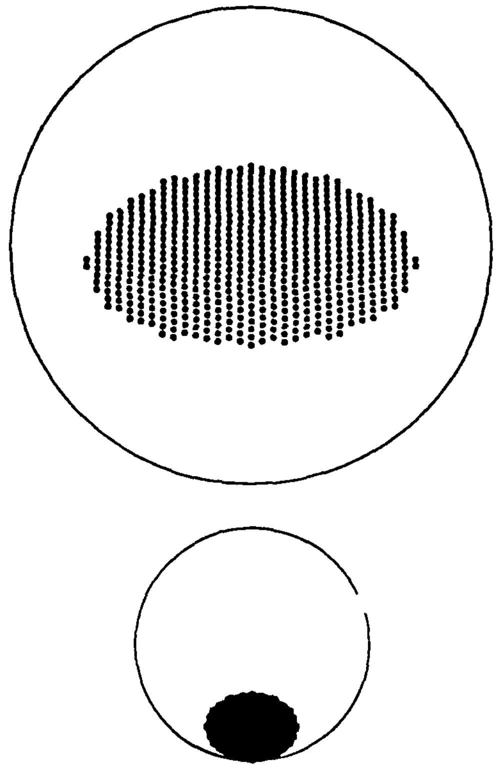

Fig. 9. Front View of Aperture Plane Ray Bundle with $\alpha = -27°$

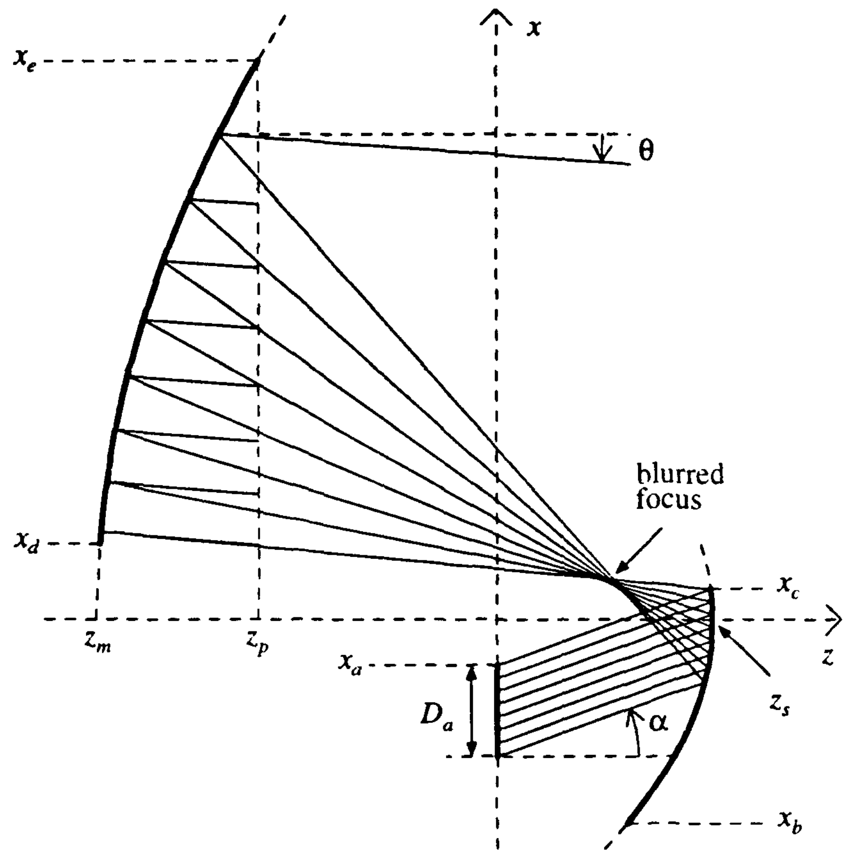

Fig. 10. Side View of Ray Bundle with $\alpha = 20°$

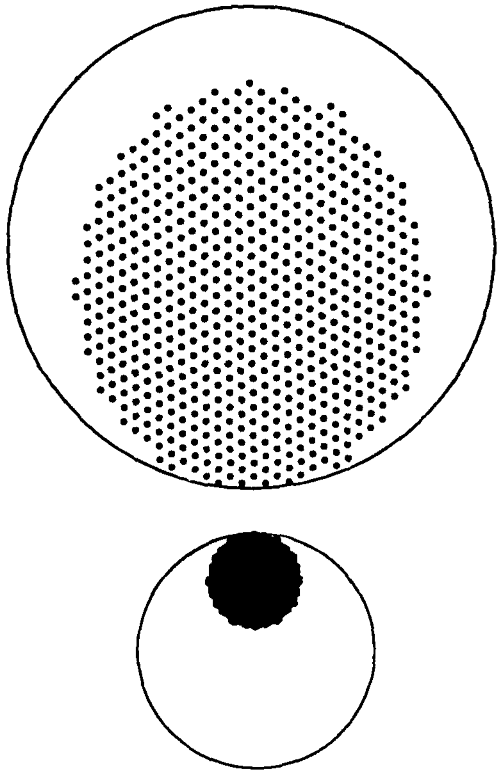

Fig. 11. Front View of Aperture Plane Ray Bundle with $\alpha = 20°$

VI. Scanning-Phase Correction

The two main types of phase errors are caused by unequal path lengths and unequal scan angles for rays that originate in the array plane and terminate in the aperture plane. Each of these errors can be eliminated by appropriately adjusting the phase on each array element. At each scan angle, a set of correction phases can be computed and used to cancel any phase distortion.

Because of the nonlinear relationship between α and θ, a uniform scan angle applied to the array plane yields a nonuniform scan angle across the main-reflector aperture plane. This gives a pointing-beam error as well as a loss in gain. It is well-known that a linear phase is required to scan a beam with no loss in power. Therefore, each ray must leave the main reflector at the same angle θ. The value of θ is derived from the small scan-angle relation given in eq. (85). Using this as a target, the nth-element array scan angle α_n lies within the range

$$\frac{F_m}{F_s}\theta - \Delta\alpha \leq \alpha_n \leq \frac{F_m}{F_s}\theta + \Delta\alpha. \tag{86}$$

A simple bisection search algorithm is performed over the interval until the element scan angle yields the desired reflector scan angle θ.

Once the scan angle correction has been made, the path length from the array plane to the aperture plane can be computed. The path length for a scan angle of $\alpha = 0°$ is constant and given by

$$P_0 = 2(z_s - z_a) + (z_a - z_m) + z_p - z_m. \tag{87}$$

As the scan angle changes, the path length for each ray changes. The path length error in radians is given by

$$\Delta_e = 2\pi\frac{(P_l - P_0)}{\lambda_0}. \tag{88}$$

This phase correction is added to each array element, so that each ray appears to have traveled the same distance.

VII. Results

Since the antenna system is designed so that there is no spillover for either reflector, the correction phase for each array-element ray should be constrained to allow no spillover. Fortunately, it turns out that the maximum correction angle for all rays for negative α is greater than $-27°$. For positive α the maximum correction angle is several degrees greater than $20°$. However, the rays that need the most correction are those with the greatest horizontal offset, so even though the corrected angle is greater than $20°$, the ray still intercepts the subreflector with no spillover.

The relationship of the design array scan angle to the reflector scan angle for the uncorrected and corrected cases is shown in Fig. 12. The reflector scan angle is defined to be the angular point of the main-beam peak. The corrected-case line has a slope of -4, equal to the negative of the magnification. The nonlinear uncorrected curve shows the increase in error as the array scan angle moves from zero. The main-beam peak power as a function of the array scan angle is shown in Fig. 13. Note that the corrected curve indicates no loss in main-beam power over the entire scan range.

These two figures indicate that the main-beam peak power is maximum and constant when the reflector scan angle is linearly related to the design array scan angle. Full correction can be achieved by forcing the reflector scan angle to be uniform. However, it should be noted that changes to the beamwidth and grating lobe separation are not restricted. The change in beamwidth and grating lobe separation could be minimized by using a shaped subreflector.

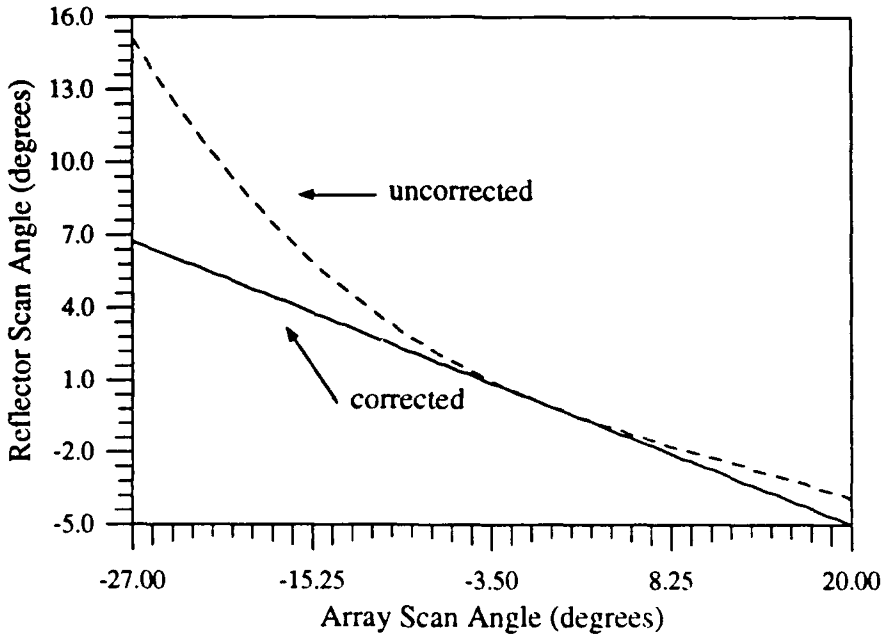

Fig. 12. Relationship of Array Scan Angle to Reflector Scan Angle

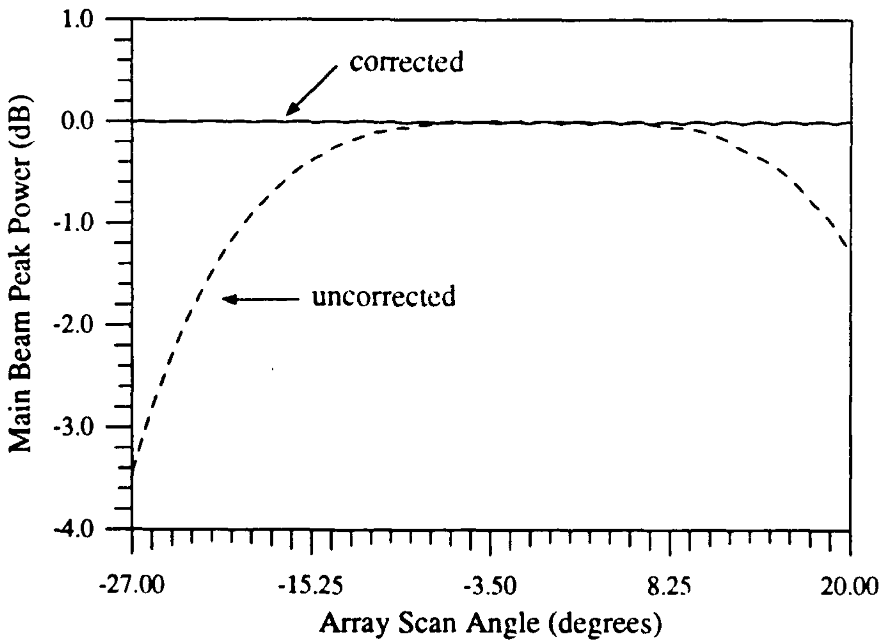

Fig. 13. Main-Beam Peak Power

VIII. Conclusion

Scanning-phase error for an offset-feed near-field Gregorian reflector can be corrected by adjusting the phase of each array element. A nonlinear array scan-angle distribution can be found that yields a linear reflector scan angle that is the same for each element beam. This phase correction can be found by a numerical search in the vicinity of the linear design angle. For the geometrical parameters considered, the phase correction can be accomplished without any loss in main-beam power or scan range.

References

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[7] Y. Rahmat-Samii, "Array Feeds for Reflector Surface Distortion Compensation: Concepts and Implementation," *IEEE Trans. Antennas and Propagat. Mag.*, vol. 32, pp. 20-26, Aug. 1990.

Appendix: Computer Program PHGREG

This appendix contains the Fortran 77 program PHGREG, as well as all the associated subroutines. The program implements the scanning phase-correction procedure described in this report. All input and output variables are defined in the comment section at the beginning of the program and each subroutine.

```
      PROGRAM PHGREG
C     ******************************************************************
C     * THIS PROGRAM SYNTHESIZES THE SCANNING PHASE OF AN OFFSET-FEED  *
C     * NEAR-FIELD GREGORIAN REFLECTOR.                                *
C     ******************************************************************
C     * TIMOTHY J. PETERS                              LAST UPDATED   *
C     * THE AEROSPACE CORPORATION                      1/15/92        *
C     * 2350 EAST EL SEGUNDO BOULEVARD.                                *
C     * EL SEGUNDO, CA 90245                                           *
C     ******************************************************************
C     * INPUTS:                                                        *
C     *                                                                *
C     *   MN - MAXIMUM ARRAY DIMENSION.                                *
C     *   DA - ARRAY DIAMETER IN WAVELENGTHS.                          *
C     *   ZA - ALONG AXIS ARRAY LOCATION IN WAVELENGTHS.               *
C     *   XA - LATERAL DISPLACEMENT OF ARRAY IN WAVELENGTHS.           *
C     *   ZS - ALONG AXIS SUBREFLECTOR LOCATION IN WAVELENGTHS.        *
C     *   XB - MINIMUM LATERAL OFFSET OF SUBREFLECTOR GEOMETRY IN      *
C     *        WAVELENGTHS.                                            *
C     *   XC - MAXIMUM LATERAL OFFSET OF SUBREFLECTOR GEOMETRY IN      *
C     *        WAVELENGTHS.                                            *
C     *   FS - FOCAL LENGTH OF SUBREFLECTOR IN WAVELENGTHS.            *
C     *   FM - FOCAL LENGTH OF MAIN REFLECTOR IN WAVELENGTHS.          *
C     *   XD - MINIMUM LATERAL OFFSET OF MAIN REFLECTOR GEOMETRY IN    *
C     *        WAVELENGTHS.                                            *
C     *   XE - MAXIMUM LATERAL OFFSET OF MAIN REFLECTOR GEOMETRY IN    *
C     *        WAVELENGTHS.                                            *
C     *   ZM - ALONG AXIS MAIN REFLECTOR LOCATION IN WAVELENGTHS.      *
C     *   NL - NUMBER OF COMPLETE HEXAGONAL FOR ARRAY GEOMETRY         *
C     *        GENERATION.                                             *
C     * ISIN - ISIN=0 MEANS NO CORRECTION IS PERFORMED AND ISIN=1      *
C     *        MEANS FULL CORRECTION OF PHASE ERROR.                   *
C     * SMIN - MINIMUM AND MAXIMU ARRAY SCAN ANGLES IN DEGREES.        *
C     * SMAX                                                           *
C     *  NSA - NUMBER OF SCAN ANGLES.                                  *
C     *  TOL - ERROR TOLERANCE FOR BISECTION ALGORITHM.                *
C     *   MI - MAXIMUM NUMBER OF ITERATIONS FOR BISECTION ALGORITHM.   *
C     *                                                                *
C     * OUTPUTS:                                                       *
C     *                                                                *
C     *   SCMIN - MINIMIM AND MAXIMUM CORRECTION ANGLES IN DEGREESFOR  *
C     *   SCMAX   FOR A GIVEN ARRAY SCAN ANGLE WRITTEN TO FILE 'RANGE'.*
C     *   RPEAK - MAIN BEAM PEAK ANGLE IN DEGREES FOR A GIVEN ARRAY    *
C     *           SCAN ANGLE WRITTEN TO FILE 'ANGLE'.                  *
C     *     PDB - MAIN BEAM PEAK POWER IN DB FOR A GIVEN ARRAY SCAN    *
C     *           ANGLE WRITTEN TO FILE 'GAIN'.                        *
C     ******************************************************************
      PARAMETER (MN=1000)
      REAL*4 X(MN),Y(MN),XT(MN),YT(MN),ASA(MN),V(MN)
```

```
      INTEGER*4 IRC(MN)
      PI=3.141593
      RAD=.17453293E-01
      OPEN(UNIT=10,FILE='RANGE')
      OPEN(UNIT=11,FILE='ANGLE')
      OPEN(UNIT=12,FILE='GAIN')
C     *****************************************************************
C     * REFLECTOR GEOMETRY INPUTS.                                    *
C     *****************************************************************
      DA=18.0
      ZA=0.0
      XA=-9.0
      ZS=42.0
      XB=-40.0
      XC=6.2
      FS=24.0
      FM=96.0
      XD=15.0
      XE=110.0
      ZM=-FM+ZS-FS
      NL=13
      ISIN=1
      SMIN=-27.0
      SMAX=20.0
      NSA=95
      TOL=0.000001
      MI=80
C     *****************************************************************
C     * GENERATE THE ARRAY ELEMENT GEOMETRY.                          *
C     *****************************************************************
      RA=DA/2.0
      S=RA/NL
      XAC=XA-DA/2.0
      YAC=0.0
      CALL GEOMET(MN,NL,S,XAC,YAC,X,Y,NE)
C     *****************************************************************
C     * COMPUTE THE PEAK POWER POINT AT BORESIGHT.                    *
C     *****************************************************************
      DO 1 I=1,NE
        ASA(I)=0.0
        V(I)=0.0
    1 CONTINUE
      CALL PEKPOW(MN,NE,X,Y,XT,YT,ASA,V,ZA,FS,ZS,FM,ZM,XB,XC,XD,XE
     &                                       ,IRC,ISIN,RPEAK,PMAXG)
C     *****************************************************************
C     * STEP THROUGH THE SCAN ANGLES.                                 *
C     *****************************************************************
      DSA=(SMAX-SMIN)/(NSA-1)
      DO 2 K=0,NSA-1
```

```
      SCAN=SMIN+K*DSA
C     ****************************************************************
C     * GENERATE THE SCANNING PHASE APPLIED TO EACH ARRAY ELEMENT.   *
C     ****************************************************************
      TARGET=-(FS/FM)*SCAN
      DO 3 I=1,NE
        IF (ISIN .EQ. 0) THEN
C         ************************************************************
C         * NO CORRECTION.                                           *
C         ************************************************************
          ASA(I)=SCAN
        ELSE
C         ************************************************************
C         * WITH CORRECTION.                                         *
C         ************************************************************
          XO=X(I)
          YO=Y(I)
          ZO=ZA
C         ************************************************************
C         * USE A BISECTION SEARCH ALGORITHM WITH +-10 DEGREES FROM  *
C         * THE TARGET SCAN ANGLE.                                   *
C         ************************************************************
          AMIN=SCAN-10.0
          AMAX=SCAN+10.0
          DO 4 J=1,MI
            A=AMIN+(AMAX-AMIN)/2.0
            CALL RAYATS(XO,YO,ZO,A,FS,ZS,FM,ZM,XE,X1,Y1,Z1
     &                                          ,X2,Y2,Z2,X3,Y3,Z3)
            T=X3-X2
            B=Z3-Z2
            THETA=180*ATAN(T/B)/PI
            ERR=THETA-TARGET
            IF ((ABS(ERR) .LT. TOL) .OR. (J .EQ. MI)) THEN
              GO TO 99
            ELSE
              IF (ERR .GT. 0.0) THEN
                AMIN=A
              ELSE
                AMAX=A
              END IF
            END IF
    4     CONTINUE
   99     CONTINUE
C         ************************************************************
C         * STORE THE COMPUTED SCAN ANGLE.                           *
C         ************************************************************
          ASA(I)=A
        END IF
    3 CONTINUE
```

```
C       ****************************************************************
C       * FIND THE MINIMUM AND MAXIMUM ARRAY SCAN ANGLES REQUIRED TO   *
C       * CORRECT THE PHASE ERROR.                                     *
C       ****************************************************************
        DO 5 I=1,NE
          IF (I .EQ. 1) THEN
            SCMIN=ASA(I)
            SCMAX=ASA(I)
          ELSE IF (ASA(I) .LT. SCMIN) THEN
            SCMIN=ASA(I)
          ELSE IF (ASA(I) .GT. SCMAX) THEN
            SCMAX=ASA(I)
          ELSE
          END IF
    5   CONTINUE
C       ****************************************************************
C       * FIND THE PEAK POWER LOCATION AND VALUE.                      *
C       ****************************************************************
        CALL PEKPOW(MN,NE,X,Y,XT,YT,ASA,V,ZA,FS,ZS,FM,ZM,XB,XC,XD,XE
     &                                       ,IRC,ISIN,RPEAK,PMAX)
C       ****************************************************************
C       * WRITE OUT THE MIN AND MAX SCAN ANGLES REQUIRED FOR EACH      *
C       * DESIGN SCAN ANGLE.                                           *
C       ****************************************************************
        WRITE(10,*) K,SCAN,SCMIN,SCMAX
C       ****************************************************************
C       * WRITE OUT THE ANGLE OF THE MAIN BEAM PEAK AT EACH SCAN ANGLE.*
C       ****************************************************************
        RPEAK=RPEAK/RAD
        WRITE(11,*) SCAN,RPEAK
C       ****************************************************************
C       * WRITE OUT THE MAIN BEAM POWER IN DB AT EACH SCAN ANGLE.      *
C       ****************************************************************
        RAT=PMAX/PMAXG
        PDB=10.0*ALOG10(RAT)
        WRITE(12,*) SCAN,PDB
    2 CONTINUE
      CLOSE(10)
      CLOSE(11)
      CLOSE(12)
      END

      SUBROUTINE GEOMET(MN,NL,S,XC,YC,X,Y,NE)
C     ******************************************************************
C     * THIS SUBROUTINE GENERATES A CIRCULAR ARRAY OF ELEMENT          *
C     * LOCATIONS BY TRUNCATING AN HEXAGONAL ARRAY.                    *
C     ******************************************************************
C     * INPUTS:                                                        *
C     *                                                                *
C     *   MN   - MAXIMUM ARRAY DIMENSION.                              *
```

```
C     *   NL    - NUMBER OF COMPLETE LAYERS OF ELEMENTS.                  *
C     *    S    - CENTER TO CENTER ELEMENT SPACING.                       *
C     *  XC,YC   - CENTER OF ARRAY.                                       *
C     *                                                                   *
C     * OUTPUTS:                                                          *
C     *                                                                   *
C     *  X(MN) - COORDINATES OF THE CENTER OF EACH ELEMENT.               *
C     *  Y(MN)                                                            *
C     *   NE    - TOTAL NUMBER OF ELEMENTS.                               *
C     *********************************************************************
      REAL*4 X(MN),Y(MN)
      REAL*8 RA,W,DP,PHI,CP,SP,T,R,SB,CB,ANGB,CX,SX,XX,YY
      RAD=.17453293E-01
      PI=.3141593E+01
C     *********************************************************************
C     * SET THE CENTER ELEMENT.                                           *
C     *********************************************************************
      X(1)=XC+0.0
      Y(1)=YC+0.0
      K=1
C     *********************************************************************
C     * LOOP THROUGH EACH LAYER.                                          *
C     *********************************************************************
      RA=NL*S
      DO 1 I=1,NL+5
        W=I*S
        DP=PI/(3*I)
C       *******************************************************************
C       * LOOP THROUGH EACH OF THE 6 QUADRANTS.                           *
C       *******************************************************************
        DO 2 L=0,5
C         *****************************************************************
C         * LOOP THROUGH EACH POINT.                                      *
C         *****************************************************************
          PHI=L*RAD*60.0
          CP=DCOS(PHI)
          SP=DSIN(PHI)
          DO 3 J=0,I-1
            T=J*S
            R=SQRT(W*W+T*T-W*T)
C           ***************************************************************
C           * COMPUTE ANGLES FOR A PURE HEXAGONAL ARRAY.                  *
C           ***************************************************************
            SB=(DSQRT(3.0D0)/2.0)*T/R
            CB=DSQRT(1.0-SB*SB)
            ANGB=DASIN((DSQRT(3.0D0)/2.0)*T/R)
            CX=DCOS(ANGB)
            SX=DSIN(ANGB)
            XX=R*CB
```

```
C
           YY=R*SB
C          ***************************************************
C          * ROTATE THE POINT.                               *
C          ***************************************************
           IF (R .LE. RA) THEN
             K=K+1
             X(K)=XC+CP*XX-SP*YY
             Y(K)=YC+SP*XX+CP*YY
           ELSE
           END IF
    3    CONTINUE
    2  CONTINUE
    1 CONTINUE
      NE=K
      RETURN
      END
```

```
      SUBROUTINE PATHLN(X0,Y0,Z0,X1,Y1,Z1,X2,Y2,Z2,X3,Y3,Z3,PL)
C     ******************************************************************
C     * THIS SUBROUTINE COMPUTES THE PATH LENGTH OF A RAY.             *
C     ******************************************************************
C     * INPUTS:                                                        *
C     *                                                                *
C     *   (X0,Y0,Z0) - COORDINATES OF POINT ON ARRAY.                  *
C     *   (X1,Y1,Z1) - COORDINATES OF POINT ON SUBREFLECTOR.           *
C     *   (X2,Y2,Z2) - COORDINATES OF POINT ON MAIN REFLECTOR.         *
C     *   (X3,Y3,Z3) - COORDINATES OF POINT IN APERTURE PLANE.         *
C     *                                                                *
C     * OUTPUT:                                                        *
C     *                                                                *
C     *   PL - PATH LENGTH.                                            *
C     ******************************************************************
      DXX=X1-X0
      DYY=Y1-Y0
      DZZ=Z1-Z0
      PL1=SQRT(DXX*DXX+DYY*DYY+DZZ*DZZ)
      DXX=X2-X1
      DYY=Y2-Y1
      DZZ=Z2-Z1
      PL2=SQRT(DXX*DXX+DYY*DYY+DZZ*DZZ)
      DXX=X3-X2
      DYY=Y3-Y2
      DZZ=Z3-Z2
      PL3=SQRT(DXX*DXX+DYY*DYY+DZZ*DZZ)
      PL=PL1+PL2+PL3
      RETURN
      END
```

```
      SUBROUTINE RAYATS(X0,Y0,Z0,AG,FS,ZS,FM,ZM,XE,X1,Y1,Z1
     &                                         ,X2,Y2,Z2,X3,Y3,Z3)
```

```
C     ******************************************************************
C     * THIS SUBROUTINE COMPUTES THE INTERSECTION OF A RAY ORIGINATING *
C     * ON THE ARRAY WITH THE SUBREFLECTOR, MAIN REFLECTOR AND THE     *
C     * MAIN REFLECTOR APERTURE PLANE.                                 *
C     ******************************************************************
C     * INPUTS:                                                        *
C     *                                                                *
C     *   XO,YO,ZO - COORDINATES OF POINT ON ARRAY.                    *
C     *   AG - SCAN ANGLE IN DEGREES.                                  *
C     *   FS - FOCAL LENGTH OF SUBREFLECTOR.                           *
C     *   ZS - ALONG AXIS POSITION OF THE SUBREFLECTOR.                *
C     *   FM - FOCAL LENGTH OF MAIN REFLECTOR.                         *
C     *   ZM - ALONG AXIS POSITION OF THE MAIN REFLECTOR.              *
C     *   XE - MAXIMUM LATERAL EXTENT OF MAIN REFLECTOR.               *
C     *                                                                *
C     * OUTPUTS:                                                       *
C     *                                                                *
C     *   X1,Y1,Z1 - COORDINATES OF POINT ON SUBREFLECTOR.             *
C     *   X2,Y2,Z2 - COORDINATES OF POINT ON MAIN REFLECTOR.           *
C     *   X3,Y3,Z3 - COORDINATES OF POINT IN APERTURE PLANE OF MAIN    *
C     *              REFLECTOR.                                        *
C     ******************************************************************
      RAD=.17453293E-01
C     ******************************************************************
C     * SET Y1=YO SINCE THE ARRAY IS SCANNED IN X-Z PLANE ONLY.        *
C     ******************************************************************
      Y1=YO
C     ******************************************************************
C     * COMPUTE X1.                                                    *
C     ******************************************************************
      ARG=RAD*AG
      TA=TAN(ARG)
      A1=TA
      B1=4.0*FS
      C1=-4.0*XO*FS+(Y1*Y1-4.0*(ZS-ZO)*FS)*TA
      IF (ABS(A1) .GT. 0.0) THEN
        ARG=B1*B1-4.0*A1*C1
        X1=(-B1+SQRT(ARG))/(2*A1)
      ELSE
        X1=XO
      END IF
C     ******************************************************************
C     * COMPUTE Z1.                                                    *
C     ******************************************************************
      Z1=ZS-(X1*X1+Y1*Y1)/(4*FS)
C     ******************************************************************
C     * COMPUTE REFLECTION MATRIX FOR THE SUBREFLECTOR.                *
C     ******************************************************************
      RO1X=(Z1-ZO)*TA
```

```
      R01Y=0.0
      R01Z=Z1-Z0
      DENOM=SQRT(X1*X1+Y1*Y1+4.0*FS*FS)
      VNX=-X1/DENOM
      VNY=-Y1/DENOM
      VNZ=-2.0*FS/DENOM
      TS11=1.0-2.0*VNX*VNX
      TS12=-2.0*VNX*VNY
      TS13=-2.0*VNX*VNZ
      TS21=TS12
      TS22=1.0-2.0*VNY*VNY
      TS23=-2.0*VNY*VNZ
      TS31=TS13
      TS32=TS23
      TS33=1.0-2.0*VNZ*VNZ
C     *****************************************************************
C     * COMPUTE THE COOEFICIENTS OF THE QUADRATIC.                    *
C     *****************************************************************
      XCOMP=TS11*R01X+TS12*R01Y+TS13*R01Z
      YCOMP=TS21*R01X+TS22*R01Y+TS23*R01Z
      ZCOMP=TS31*R01X+TS32*R01Y+TS33*R01Z
C     *****************************************************************
C     * COMPUTE THE POINT WHERE THE RAY REFLECTED OFF THE SUBREFLECTOR *
C     * INTERCEPTS THE MAIN REFLECTOR.                                *
C     *****************************************************************
      U1=YCOMP/XCOMP
      U2=ZCOMP/XCOMP
      A2=1.0+U1*U1
      B2=2.0*U1*(Y1-U1*X1)-4.0*U2*FM
      C2=(Y1-U1*X1)*(Y1-U1*X1)+4.0*FM*(ZM-Z1+U2*X1)
      IF ((ABS(ALPHA).EQ. 0.0) .AND.(ABS(X0).EQ. 0.0)
     &                 .AND.(ABS(Y0) .EQ. 0.0))  THEN
        X2=X1
      ELSE
        ARG=B2*B2-4*A2*C2
        X2A=(-B2-SQRT(ARG))/(2*A2)
        X2B=(-B2+SQRT(ARG))/(2*A2)
        Y2A=Y1+(X2A-X1)*U1
        IF (Y2A-Y1 .GT. 0.0) THEN
          P1=1.0
        ELSE
          P1=-1.0
        END IF
        IF (YCOMP .GT. 0.0) THEN
          P2=1.0
        ELSE
          P2=-1.0
        END IF
        IF (X2A-X1 .GT. 0.0) THEN
```

```
          P3=1.0
        ELSE
          P3=-1.0
        END IF
        IF (XCOMP .GT. 0.0) THEN
          P4=1.0
        ELSE
          P4=-1.0
        END IF
        IF ((P1 .EQ. P2).AND.(P3 .EQ. P4)) THEN
          X2=X2A
        ELSE
          X2=X2B
        END IF
      END IF
      Y2=Y1+(X2-X1)*U1
      Z2=(X2*X2+Y2*Y2)/(4*FM)+ZM
C     ******************************************************************
C     * COMPUTE REFLECTION MATRIX FOR THE MAIN REFLECTOR.              *
C     ******************************************************************
      R12X=X2-X1
      R12Y=Y2-Y1
      R12Z=Z2-Z1
      DENOM=SQRT(X2*X2+Y2*Y2+4.0*FM*FM)
      VNX=-X2/DENOM
      VNY=-Y2/DENOM
      VNZ=2.0*FM/DENOM
      TM11=1.0-2.0*VNX*VNX
      TM12=-2.0*VNX*VNY
      TM13=-2.0*VNX*VNZ
      TM21=TM12
      TM22=1.0-2.0*VNY*VNY
      TM23=-2.0*VNY*VNZ
      TM31=TM13
      TM32=TM23
      TM33=1.0-2.0*VNZ*VNZ
C     ******************************************************************
C     * COMPUTE THE COORDINATES IN THE APERTURE PLANE OF THE MAIN      *
C     * REFLECTOR.                                                     *
C     ******************************************************************
      DENOM=TM31*R12X+TM32*R12Y+TM33*R12Z
      V1=(TM21*R12X+TM22*R12Y+TM23*R12Z)/DENOM
      V2=(TM11*R12X+TM12*R12Y+TM13*R12Z)/DENOM
      Z3=ZM+XE*XE/(4.0*FM)
      Y3=Y2+(Z3-Z2)*V1
      X3=X2+(Z3-Z2)*V2
      RETURN
      END

      SUBROUTINE PEKPOW(MN,NE,X,Y,XT,YT,ASA,V,ZA,FS,ZS,FM,ZM,XB,XC,XD
```

```
     &                                          ,XE,IRC,ISIN,RPEAK,PMAX)
C     *****************************************************************
C     * THIS SUBROUTINE COMPUTES THE PEAK POWER VALUE AND THE ANGULAR *
C     * LOCATION.                                                     *
C     *****************************************************************
C     * INPUTS:                                                       *
C     *                                                               *
C     *   MN - MAXIMUM ARRAY DIMENSION.                               *
C     *   NE - TOTAL NUMBER OF ELEMENTS.                              *
C     *  X(MN) - COORDINATES OF THE CENTER OF EACH ELEMENT.           *
C     *  Y(MN)                                                        *
C     *  XT(MN) - COORDINATES OF THE CENTER OF EACH ELEMENT PROJECTED *
C     *  YT(MN)   ONTO THE MAIN REFLECTOR APERTURE PLANE.             *
C     *  ASA(MN) -  ARRAY OF CORRECTION PHASE ANGLES IN DEGREES.      *
C     *   V(MN) -  ARRAY OF TOTAL PHASE CORRECTION INCLUDING PATH     *
C     *            LENGTH ERROR CORRECTION.                           *
C     *   ZA - ALONG AXIS ARRAY LOCATION IN WAVELENGTHS.              *
C     *   FS - FOCAL LENGTH OF SUBREFLECTOR IN WAVELENGTHS.           *
C     *   ZS - ALONG AXIS SUBREFLECTOR LOCATION IN WAVELENGTHS.       *
C     *   FM - FOCAL LENGTH OF MAIN REFLECTOR IN WAVELENGTHS.         *
C     *   ZM - ALONG AXIS MAIN REFLECTOR LOCATION IN WAVELENGTHS.     *
C     *   XB - MINIMUM LATERAL OFFSET OF SUBREFLECTOR GEOMETRY IN     *
C     *        WAVELENGTHS.                                           *
C     *   XC - MAXIMUM LATERAL OFFSET OF SUBREFLECTOR GEOMETRY IN     *
C     *        WAVELENGTHS.                                           *
C     *   XD - MINIMUM LATERAL OFFSET OF MAIN REFLECTOR GEOMETRY IN   *
C     *        WAVELENGTHS.                                           *
C     *   XE - MAXIMUM LATERAL OFFSET OF MAIN REFLECTOR GEOMETRY IN   *
C     *        WAVELENGTHS.                                           *
C     *  IRC(MN) -  INTEGER ARRAY USED TO MARK ANY RAYS WHICH SPILL   *
C     *             OVER THE SUBREFLECTOR.                            *
C     * ISIN - ISIN=0 MEANS NO CORRECTION IS PERFORMED AND ISIN=1     *
C     *        MEANS FULL CORRECTION OF PHASE ERROR.                  *
C     *                                                               *
C     * OUTPUTS:                                                      *
C     *                                                               *
C     * RPEAK - PEAK POWER ANGULAR LOCATION IN DEGREES.               *
C     *  PMAX - PEAK POWER LEVEL.                                     *
C     *****************************************************************
      REAL*4 X(MN),Y(MN),XT(MN),YT(MN),ASA(MN),V(MN)
      INTEGER*4 IRC(MN)
      PI=3.141593
      TP=.62831853E+01
      RAD=.17453293E-01
C     *****************************************************************
C     * TRANSFORM ALL ARRAY COORDINATES TO THE MAIN REFLECTOR APERTURE *
C     * PLANE.                                                        *
C     *****************************************************************
      ZAP=ZM+(1/(4*FM))*XE*XE
```

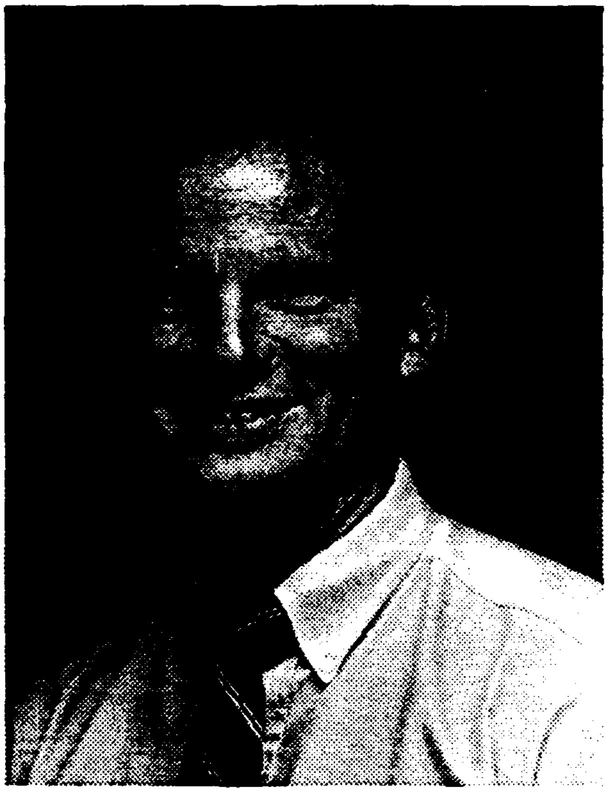
```
      PLO=2*ABS(ZS-ZA)+ABS(ZA-ZM)+ABS(ZAP-ZM)
      XCS=(XB+XC)/2.0
      SRADUS=ABS(XC-XCS)
      DO 1 I=1,NE
        XO=X(I)
        YO=Y(I)
        ZO=ZA
        CALL RAYATS(XO,YO,ZO,ASA(I),FS,ZS,FM,ZM,XE,X1,Y1,Z1
     &                                         ,X2,Y2,Z2,X3,Y3,Z3)
C       ****************************************************************
C       * CHECK TO SEE IF THE RAY MISSES THE SUBREFLECTOR OR THE MAIN  *
C       * REFLECTOR.                                                   *
C       ****************************************************************
        DX=X1-XCS
        RAY=SQRT(DX*DX+Y1*Y1)
        IF (RAY .LE. SRADUS) THEN
          IRC(I)=0
        ELSE
          IRC(I)=1
        END IF
C       ****************************************************************
C       * STORE THE APERTURE COORDINATES.                              *
C       ****************************************************************
        XT(I)=X3
        YT(I)=Y3
C       ****************************************************************
C       * COMPUTE AND STORE THE APERTURE SCAN ANGLE.                   *
C       ****************************************************************
        T=X3-X2
        B=Z3-Z2
        IF (T .EQ. 0.0) THEN
          ARG=0.0
        ELSE
          ARG=T/B
        END IF
        ASCAN=ATAN(ARG)
C       ****************************************************************
C       * COMPUTE THE PATH LENGTH ERROR.                               *
C       ****************************************************************
        CALL PATHLN(XO,YO,ZO,X1,Y1,Z1,X2,Y2,Z2,X3,Y3,Z3,PL)
C       ****************************************************************
C       * GENERATE THE PATH LENGTH PHASE.                              *
C       ****************************************************************
        PLP=TP*PL
        PLE=PL-PLO
C       ****************************************************************
C       * GENERATE THE SCAN PHASE.                                     *
C       ****************************************************************
        SCPH=-TP*SIN(ASCAN)*XT(I)
```

```
C         ****************************************************************
C         * GENERATE THE PATH LENGTH CORRECTION PHASE.                   *
C         ****************************************************************
          PLCP=-TP*PLE
C         ****************************************************************
C         * GENERATE THE ELEMENT PHASE.                                  *
C         ****************************************************************
          IF (ISIN .EQ. 0) THEN
            V(I)=SCPH+PLP
          ELSE
            V(I)=SCPH+PLP+PLCP
          END IF
    1   CONTINUE
C       ******************************************************************
C       * SEARCH RANGE LIMITS IN RADIANS.                                *
C       ******************************************************************
        A=-RAD*10.0
        B=RAD*17.0
C       ******************************************************************
C       * FIND THE PEAK OF THE MAIN BEAM.                                *
C       ******************************************************************
        NMAX=378
        R=0.0
        P=0.0
        DR=(B-A)/NMAX
        PMAX=-1.0
        RPEAK=0.0
        DO 2 I=0,NMAX-1
          R=A+I*DR
          CALL AFPAT(MN,NE,XT,YT,V,R,P,PAT)
          IF (PAT .GT. PMAX) THEN
            PMAX=PAT
            RPEAK=R
          ELSE
          END IF
    2   CONTINUE
        RETURN
        END

        SUBROUTINE AFPAT(MN,NE,X,Y,V,T,P,PAT)
C       ******************************************************************
C       * THIS SUBROUTINE COMPUTES THE SQUARE OF THE ARRAY FACTOR.       *
C       ******************************************************************
C       * INPUTS:                                                        *
C       *                                                                *
C       *   MN - MAXIMUM ARRAY DIMENSION.                                *
C       *   NE - TOTAL NUMBER OF ELEMENTS.                               *
C       *  X(MN) - COORDINATES OF THE CENTER OF EACH ELEMENT.            *
C       *  Y(MN)                                                         *
C       *  V(MN) -  ARRAY OF TOTAL PHASE CORRECTION INCLUDING PATH       *
```

```
C     *          LENGTH ERROR CORRECTION.                                *
C     *    T - THETA IN RADIANS.                                         *
C     *    P - PHI IN RADIANS.                                           *
C     *                                                                  *
C     * OUTPUT:                                                          *
C     *                                                                  *
C     *   PAT - SQUARE OF ARRAY FACTOR.                                  *
C     ********************************************************************
      REAL*4 X(MN),Y(MN),V(MN),KX,KY
      TP=.62831853E+01
      CP=COS(P)
      SP=SIN(P)
      ST=SIN(T)
      KX=TP*ST*CP
      KY=TP*ST*SP
      SUM1=0.0
      SUM2=0.0
      DO 1 I=1,NE
        ARG=V(I)+KX*X(I)+KY*Y(I)
        SUM1=SUM1+COS(ARG)
        SUM2=SUM2+SIN(ARG)
    1 CONTINUE
      PAT=SUM1*SUM1+SUM2*SUM2
      RETURN
      END
```

About the Author



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